MULTIUSER DIVERSITY FOR MIMO WIRELESS SYSTEMS WITH LINEAR RECEIVERS

Robert W. Heath Jr.[1] *Manish Airy*[2] *Arogyaswami J. Paulraj*[1]

[1]Information Systems Laboratory
Stanford University
Stanford, CA 94305-9510
{rheath,apaulraj}@stanford.edu

[2]Iospan Wireless Inc.
3099 N. First Street
San Jose, CA 95134
mairy@iospanwireless.com

ABSTRACT

MIMO communication links, i.e. those with multiple transmit and receive antennas, offer significant advantages in terms of rate and reliability. In cellular systems, however, gains may be limited due to fading and interference. One potential solution is known as multiuser diveristy, in which a packet scheduler improves throughput by exploiting the independence of the fading and interference statistics of different users. In this paper, we consider the problem of exploiting multiuser diversity in MIMO systems, especially those with zero-forcing linear receivers. We propose a number of different scheduling disciplines and compare them in terms of average throughput as a function of the number of users and number of antennas.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) communication systems, i.e., systems that employ communication links with multiple transmit and receive antennas, offer significant advantages in terms of rate and reliability. These gains can be realized by exploiting spatial multiplexing [1] in which data is multiplexed across the transmit antennas. In cellular systems, however, the gain from spatial multiplexing is limited by fading and interference.

Motivated by the information theoretic results in [2, 3], one approach to increase the throughput of multi-user systems is to use multiuser diversity to take advantage of the independence of the fading statistics of different users[4]. In a multiuser diversity system, a packet scheduler uses instantaneous knowledge of the signal-to-noise ratio (SNR) of each to user to allocate resources to the user with the based on channel quality. Multiuser diversity is appropriate for high-speed data transmission since data packets are more tolerant to scheduling delays than constant bit rate services such as voice. While offering impressive performance improvements, previous work on multiuser diversity has not fully explored the application of multiuser diversity to spatial multiplexing systems.

In this paper we study the improvement in average throughput due to multiuser diversity in MIMO cellular systems that employ spatial multiplexing in flat-fading channels. While in the single antenna scenario the attenuation of the channel plays a role, in the MIMO case we show that the singular values of the channel better characterize performance. Optimal receivers are difficult to implement therefore we pay particular attention to zero-forcing (ZF) linear receivers. We argue that for a large number of users the ZF receiver approaches the average performance of the optimal receiver since the multiuser diversity effect can compensate for poorly conditioned matrix channels. For the ZF receiver we propose a scheduling algorithm that assigns all the transmit antennas to a single user and compare with an algorithm that assigns the transmit antennas independently. We show that assigning the transmit antennas independently offers significant throughput improvements at the penalty of an increase in the feedback required. We use results from [5] to study the asymptotic behavior for a large number of users.

Scheduling for MIMO cellular systems was considered in [6] for MIMO systems with either transmit diversity or with spatial multiplexing systems with maximum likelihood (ML) receivers [7]. Compared with [6] we consider spatial multiplexing with both optimal and practically relevant linear receivers. We show that there may be significant benefits to not assigning all available antennas to a single user. A technique for scheduling with multiple transmit antennas known as opportunistic beamforming has been considered in [5]. Therein the transmitter cycles through a sequence of beam patterns to induce more variations in an equivalent scalar channel. The emphasis of [5] is to convert the vector channel to a scalar channel, improving the operation of scalar multiuser diversity, and the proposed technique does not obtain the benefits of the multiple modes induced by the MIMO system.

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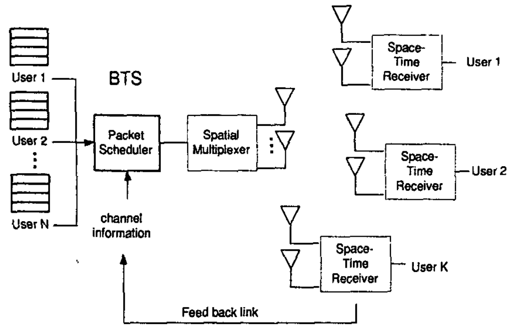

Fig. 1. Downlink multiuser diversity with MIMO communication links.

This paper is organized as follows. In Section 2 we introduce multiuser diversity in MIMO systems and in Section 3 we discuss average throughput. In Section 4 we propose two scheduling algorithms for MIMO systems with ZF receivers. In Section 5 we examine asymptotic performance in Rayleigh MIMO channels. Section 6 contains some performance simulations while Section 7 contains our conclusions.

2. PROBLEM FORMULATION

A simple model for the downlink of a wireless communication system is illustrated in Fig. 1. There is a single base transceiver station (BTS) communicating with K users. The BTS has M_t transmit antennas while each user has M_r receive antennas. A queue of packets is stored at the BTS for each of the K users. Based on feedback of the channel state from each of the users, the BTS chooses the user(s) to which it will send the next packet(s).

There are a number of different ways that the BTS can transmit information on a MIMO communication link. One example is known as space-time codes, e.g. [8, 9], in which the antennas are use purely for diversity advantage. Another example is spatial multiplexing [1] [10] in which independent streams of information are transmitted over the different transmit antennas. There are also approaches that bridge the two, e.g. linear dispersion codes [11] [10], that have rate and diversity advantage. The presence of multiple users provides an effective form of diversity advantage therefore we focus on spatial multiplexing that offers high peak data rates and reasonable decoding complexity.

Consider codeword blocks of duration N symbol periods. The baseband block fading channel model is described by

$$\mathbf{y}_k(t,n) = \sqrt{\frac{E_s}{M_t}}\mathbf{H}_k(t)\mathbf{x}(t,n) + \mathbf{v}_k(t,n) \quad (1)$$

$$k = 0, 1, \ldots, K-1 \text{ and } n = 0, 1, \ldots, N-1$$

where $\mathbf{x}(t,n)$ is the complex $M_t \times 1$ transmitted symbol vector in time-slot t, symbol period n, $\mathbf{y}(t,n)$ is the corresponding complex $M_r \times 1$ received symbol vector, $\mathbf{H}_k(t)$ is the channel matrix from the transmit array to the receive array of user k, and $\{\mathbf{v}_k(t,n)\}_{t,n}$ is a sequence of independent identially distributed circularly symmetric complex Gaussian random vectors with distribution $\mathcal{CN}(0, N_o/2\mathbf{I}_{M_r})$.

Throughout this paper we assume that the bandwidth of the signal is much less than the coherence time of the channel so that it is frequency flat. Additionally, we suppose that the channel is constant over the time slot of length N but varies from time slot to time slot.

The spatial multiplexer constructs the transmitted signal vector by multiplexing M_t independent data streams [1, 12]. The data streams could come from the queue of a single user or from the queues of different users. The power constraint is chosen such that[1] $\mathcal{E}\{\mathbf{x}_m(t,n)\mathbf{x}_m^H(t,n)\} = 1$. Thus the *power per transmit antenna is fixed*. For the duration of this paper we consider $M_r = M_t$ which is typical in spatial multiplexing systems.

In downlink multiuser diversity systems the goal is to maximize the sum capacity defined as the maximum achievable sum of long-term average data rates transmitted to all users [5]. In the single-input single-output (SISO) case it has been shown that transmitting to the user with the strongest channel in the given time slot is a strategy that can achieve this capacity [3]. Therefore in SISO systems it suffices for the receivers to feed back to the transmitter their instantaneous SNR. The transmitter then selects the user with the best SNR for transmission.

The challenge of multiuser diversity for MIMO systems is that the link throughput depends on both the received SNR as well as the subspace structure of the channel matrix and the receiver [13]. Further, analagous results to [3] for the downlink MIMO broadcast channel do not seem to be available thus the optimal strategy for allocating antennas and time slots to users is unknown.

In this paper we design scheduling algorithms for multiuser MIMO systems in which the BTS attempts to maximize the sum of the average data rate in the system. We assume that each receiver conveys one or more performance metrics to the BTS but not necessarily complete channel state information. We consider two antenna assignment methodologies at the BTS. In one case, the BTS allocates a single time slot and all transmit antennas to a single user. In the other case, the BTS allocates the transmit antennas independently. Therefore it may happen that a user receives

[1] We use $\mathcal{E}$ for expectation, ^{H} for conjugate transpose, and $\mathbf{x}_m$ to denote the m^{th} component of vector $\mathbf{x}$.

transmissions from none, one, or more of the transmit antennas.

In real systems, fairness and delay are also considered by the packet scheduling algorithm. This strategy is required since users may see channels with different statistics. For example the mean receive signal level may be greater for users that are closer to the BTS than for those who are further away. A good discussion of a fair scheduling algorithm for extracting multiuser diversity has been proposed [4] and implemented in the IS-856 system [14]. In this paper, since we are dealing with the presence of multiple transmit antennas, we assume that the statistics are the same for each user. For this case the max scheduler is also a fair scheduler. We leave fairness issues in asymmetric channels and delay constraints for future work.

3. THROUGHPUT IN MULTIPLEXING SYSTEMS

The key to scheduling packets in the multiuser diversity system is to define the notion of a *good* channel. This is enabled by understanding the relationship between the channel and the data rate achievable for a given realization of the channel $\mathbf{H}_k(t)$.

In a practical system the rate supported on a communication link depends on the target bit error rate which in turn depends on the application. The target bit error rate for a spatial multiplexing system as a function of the channel is difficult to capture in closed form [7]. Therefore in this paper we will use the capacity, i.e. the maximum achievable error rate for an arbitrarily low probability of error for a given channel, to map a channel to a throughput.

In this section we assume that we assign all the transmit antennas to a single user. The capacity of a single-user AWGN MIMO system with channel $\mathbf{H}_k(t)$ is given by [15]

$$C_k(t) := \log\det\left(\mathbf{I}_{M_r} + \frac{E_s}{N_o M_t}\mathbf{H}_k(t)\mathbf{H}_k^H(t)\right) \quad (2)$$

$$= \sum_{m=0}^{M_t} \log\left(1 + \frac{E_s}{N_o M_t}\lambda_m(\mathbf{H}_k(t))\right) \quad (3)$$

where $\lambda_m(\mathbf{A})$ is the m^{th} eigenvalue of $\mathbf{A}\mathbf{A}^H$. We refer to this as the link capacity with an optimal receiver.

Note that if $M_t = 1$ and/or $M_r = 1$, then (2) simplifies to

$$C_k(t) = \log\left(1 + \frac{SNR}{M_t}\|\mathbf{H}_k(t)\|_F^2\right).$$

When the channel is SIMO or MISO the throughput is simply a function of the power contained in the channel capture by $\|\mathbf{H}_k(t)\|_F^2$. In the MIMO case, however, the throughput is instead a function of each of the eigenvalues values of the channel. Thus for MIMO channels throughput depends on both received power (through the sum of the eigenvalues) and the subspace structure of the channel.

In spatial multiplexing systems with optimal receivers, e.g. the maximum likelihood receiver, the decoding complexity may be prohibitive for higher spectrum efficiencies. Therefore in practical systems, suboptimal receivers like the zero-forcing receiver that simply invert the channel and detect the streams independently may be preferred to reduce receiver complexity.

Let $\mathbf{G}_k(t) := \mathbf{H}_k(t)^\dagger$ (where $\dagger$ is the Moore-Penrose pseudo-inverse). Applying $\mathbf{G}_k(t)$ to (1)

$$\hat{\mathbf{x}}(t,n) = \mathbf{x}(t,n) + \mathbf{G}_k(t)\mathbf{v}(t,n) \quad (4)$$

and it is clear that noise term is colored. Decoding as if the noise was independent, the pseudo-inverse converts the system to a series of M_t parallel channels with received SNR $E_s/N_o M_t[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}$. Therefore the m^{th} stream for user k is given by

$$\mathbf{y}_{k,m}(t,n) = \mathbf{x}_m(t,n) + \mathbf{v}_{k,m}(t,n) \quad (5)$$

where $\{\mathbf{v}_{k,m}(t,n)\}_{t,n}$ is a complex Gaussian circularly symmetric random sequence with $\mathcal{CN}(0, N_o[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}M_t)$.

The capacity of the set of AWGN parallel channels in (5) is then

$$C_{ZF,k}(t) = \sum_{m=0}^{M_t-1}\log\left(1 + \frac{E_s}{N_o M_t}\frac{1}{[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}}\right).$$

It is known that in general $C_{ZF,k}(t) \leq C_k(t)$ since the ZF receiver colors the noise and we assume the noise is spatially white in the decoding. Equality occurs when $\mathbf{H}_k(t)^H\mathbf{H}_k(t)/\|\mathbf{H}_k(t)\|_F^2 = \mathbf{I}_{M_t}$ thus $\mathbf{H}_k(t)$ has columns that are orthogonal.

4. SCHEDULING ALGORITHMS

In this section we propose a number of scheduling algorithms for MIMO spatial multiplexing systems.

4.1. Selecting the Best User for the Optimal Receiver

Assuming that all transmit antennas are dedicated for a single user, the maximum capacity scheduler chooses k such that $k = \arg\max_k C_k(t)$. The corresponding throughput for this scheduling discipline is naturally

$$C(t) := \max_k C_k(t). \quad (6)$$

Consider a group of users for which $\|\mathbf{H}_k(t)\|_F^2$ is fixed. Using the concavity of the log function

$$C_k(t) \leq \log\left(1 + \frac{E_s\|\mathbf{H}_k(t)\|_F^2}{N_o M_t}\sum_{m=0}^{M_t}\frac{\lambda_m(\mathbf{H}_k(t))}{\|\mathbf{H}_k(t)\|_F^2}\right) (7)$$

which holds with equality when $\lambda_m(\mathbf{H}_k(t)) = 1$. Equivalently, this occurs when $\mathbf{H}_k(t)^H\mathbf{H}_k(t)/||\mathbf{H}_k(t)||_F^2 = \mathbf{I}_{M_t}$ thus $\mathbf{H}_k(t)$ has columns that are orthogonal. For a group of users with similar $||\mathbf{H}_k(t)||_F^2$ the user that has the best throughput will have the channel that is closest to orthogonal. Dividing the possible $||\mathbf{H}_k(t)||_F^2$ into a sufficiently large number of bins, with enough users, the user in the largest received SNR bin with the most orthogonal channel will be chosen as optimal. Good channels in MIMO systems it seems have a large $||\mathbf{H}_k(t)||_F^2$ and are close to orthogonal.

4.2. Selecting the Best User for the ZF Receiver

For a large number of users, the multiuser diversity effect should compensate for the use of the suboptimal ZF receiver. Therefore we now focus on strategies for selecting the optimal user based on the criterion that maximizes the ZF link capacity. Again we assume that all M_t transmit antennas are dedicated to a single user. The max link capacity scheduler for the ZF case chooses the user that solves $k = \arg\max_k C_{ZF,k}(t)$. The corresponding throughput achieved with this strategy is

$$C_{ZF,k}(t) = \sum_{m=0}^{M_t-1} \log\left(1 + \frac{E_s}{N_o M_t}\frac{1}{[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}}\right)$$

This strategy requires that each user feed back quantized versions of $C_{ZF,k}(t)$ as opposed to quantized versions of $\mathbf{H}_k(t)$. This is reasonable since we do not assume full channel knowledge at the transmitter.

In the limit as the number of users grows large, it appears that the optimal solution for maximizing $C_{ZF}(t)$ will approach that of $C)(t)$ since the best channels in both cases have orthogonal columns with a large Frobenius norm. This motivates using the *suboptimal linear receiver and relying on multiuser diversity to select the user with the 'most invertible' channel.* Of course, since we will deal with a finite number of users the difference between $C(t)$ and $C_{ZF}(t)$ will be nonnegligable. We examine this case in the simulations.

4.3. Suboptimal ZF Selection Criterion

Motivated by results in [13] a suboptimal selection criterion is to use the SNR of the worst stream, as determined by $\min_m [\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}$ to choose the best user. We refer to the scheduler that selects based on this criterion as the max-min scheduler. Like scalar multiuser diversity systems, the receiver provide the BTS with a single value that captures the quality of the channel. Since we find that good channels satisfy $[\mathbf{H}_r^H(t)\mathbf{H}_r(t)]_{mm}^{-1}$ is constant and as large as possible, the proposed metric should converge to the solution in (7) as the number of users increases.

The max-min scheduler chooses the user that satisfies $k = \arg\max_k \min_m 1/[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}$. Without knowledge of the other values of $[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}$, the rate achieved by the max-min scheduler in a given time slot is given by

$$C_{max-min}(t) \geq M_t \log\left(1 + \frac{E_s}{N_o M_t}\alpha\right) \quad (8)$$

where $\alpha = \max_k \min_m 1/[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}$. Equality holds when the best user has a channel with orthogonal columns. The lower bound on the achievable rate in this case is simply a function of the user with the best minimum SNR. At worst, the max-min scheduler retains the MIMO improvement (factor of M_t in (8)) at the penalty of a loss in SNR. It follows without proof that $C_{ZF}(t) \geq C_{max-min}(t)$

4.4. Selecting the Best M_t Streams for the ZF Receiver

In the previous section we assumed that each user employed spatial multiplexing and thus the user chosen was allocated all of the available M_t transmit antennas. The ZF receiver coupled with suboptimal detection, however, transforms the $M_r \times M_t$ AWGN channel into a series of M_t parallel AWGN channels. Due to the independence of the parallel channels there is no need to assign all the channels to a single user. Instead all users can *compete independently for each transmit antenna.* A user may be assigned zero, one, or more antennas based on their current channel state. This naturally generalizes SISO multiuser diversity algorithms in [4]. We refer to the scheduler that allocates antennas in an independent manner as the independent stream scheduler.

For antenna m, the independent stream scheduler transmits a packet from the user that satisfies

$$k = \arg\max_k \log\left(1 + \frac{E_s}{N_o M_t}\frac{1}{[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}}\right).$$

The maximum rate achieved by the independent stream scheduler is

$$C_{ind}(t) = \sum_{m=0}^{M_t-1} \log\left(1 + \frac{E_s}{N_o M_t}\max_k \frac{1}{[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}}\right).$$

A simple argument shows that $C_{ind}(t) \geq C_{ZF}(t) \geq C_{max-min}(t)$ since assigning all the antennas to one user is a special case of the independent stream scheduler.

5. RAYLEIGH MIMO PERFORMANCE

Now suppose that the channel gains $[\mathbf{H}_k(t)]_{l,m}$ are independent identically distributed circularly symmetric complex Gaussian random variables with distribution $\mathcal{CN}(0,1)$

uncorrelated in space and in time. This is the MIMO Rayleigh fading channel model that is appropriate when there is significant scattering in the environment and the antenna spacing at the BTS and at each receiver are much greater than the coherence distance. MIMO Rayleigh fading is a standard assumption in the design of space-time codes [8] [11] [10].

Under the MIMO Rayleigh fading model, the performance of the system is a function of the distribution of the SNR of each substream. Let $\gamma := \sqrt{E_s/M_t N_o}$ and let $\gamma_m := \gamma_0/[\mathbf{H}_k^H(t)\mathbf{H}_k(t)]_{mm}^{-1}$. We have shown that [16] the distribution of the SNR on the m^{th} stream for $M_t = M_r$ is

$$f(\gamma_{m,k}) \quad = \quad \gamma^{-1} e^{-\gamma_{m,k}\gamma^{-1}} \tag{9}$$

which is simply a scaled version of the chi-squared distribution with two degrees of freedom. The average SNR is γ. We show that the distribution is independent of the stream.

For spatial multiplexing with a zero-forcing receiver we propose to choose the optimal user based only on the smallest substream $\min_m \gamma_{m,k}$. Using order statistics, it is easy to show that the distribution for the smallest SNR is again chi-squared with

$$f(\gamma_{min,k}) \quad = M_t \gamma^{-1} e^{-M_t \gamma_{min,k}\gamma^{-1}} \tag{10}$$

Comparing with (9) in this case the average SNR is γ/M_t and there is an additional loss in the average SNR by a factor of M_t.

Insight into the performance gain to due to multiuser diversity can be studied by asymptotic analysis assuming that the number of users grows large. In these cases we are interested in $\gamma_{min,K} := \max_k \gamma_{min,k}$ and $\gamma_{m,K} := \max_k \gamma_{m,k}$.

An analysis for a large number of users is provided in [5] and we can readily apply the results to this case. Due to space-constraints we state these results without proof.

Proposition 1 *For the max-min diversity scheduler,* $\gamma_{min,K}$ *grows as* $\gamma/M_t \log K$ *for large numbers of users. For the independent stream scheduler,* $\gamma_{m,K}$ *grows as* $\gamma \log K$ *for large numbers of users.*
Proof: Follows from [5, Lemma 2].

The gain from multiuser diversity is the $\log K$. The primary difference in performance is that the max-min scheduler suffers an additional penalty of M_t since it decides based only on the worst substream. Since $C_{max-min}(t) \leq C_{ZF(t)} \leq C(t)$ this shows that we should expect multiuser diversity gain in MIMO spatial multiplexing systems.

6. MULTIUSER DIVERSITY COMPARISON

In this section we examine the performance of various proposed multi-user diversity techniques in the $M \times M$ Rayleigh MIMO channel for $M = 1, 2, 3, 4$. We plot the average throughput for different scheduling algorithms as a function of the number of users. The curves from bottom to top increase with the number of antennas.

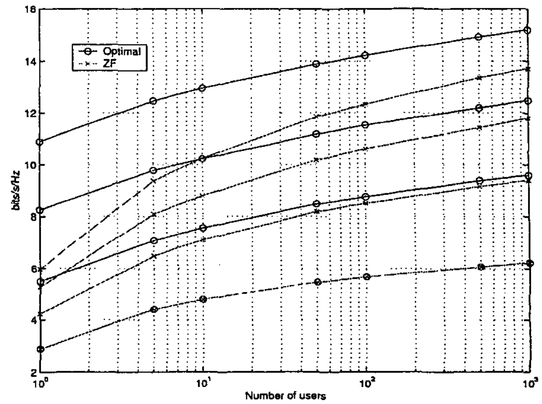

Fig. 2. Spatial multiplexing: optimal versus ZF scheduler

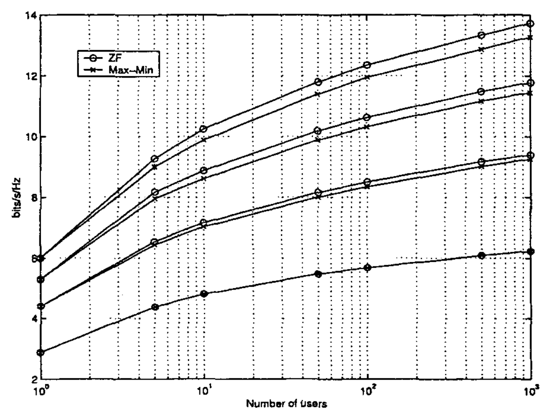

Fig. 3. ZF: max capacity scheduler versus max-min scheduler

First we compare the performance the maximum capacity scheduling disciplines that assign all the transmit antennas to a single user for the optimal and the ZF receiver. In Fig. 2 we compare the average throughput using optimal scheduling with that obtained using the ZF receiver. For each antenna configuration we see that the ZF scheduler approaches the max capacity scheduler for a large numbers of users. Compared with the single user case, equivalent to round-robin scheduling, both schemes offer substantial multiuser diversity improvements. With a sufficient number of users, the multiuser diversity benefit allows the ZF receiver to outperform the optimal receiver with no multiuser diversity advantage.

Now for the ZF receiver we explore the loss due to scheduling based on the worst substream. As illustrated

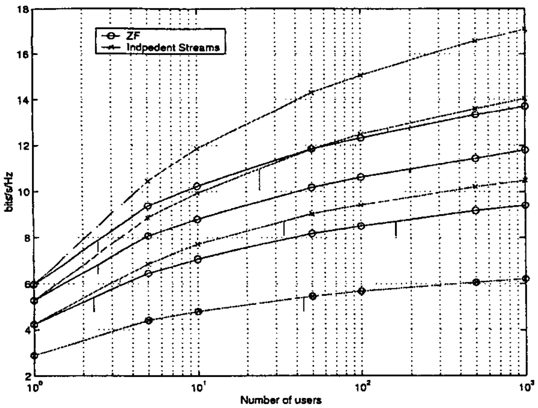

Fig. 4. ZF: max capacity scheduler versus independent stream scheduler

in Fig. 3, both scheduling algorithms exhibit multiuser diversity benefit of the case of $K = 1$. The throughput loss by scheduling based on the worst substream is minimal.

Finally we compare the average throughput achieved by the ZF max capacity scheduler with that of the independent stream scheduler. As illustrated in Fig. 4 there is substantial improvement in the average throughput by allowing antennas to be assigned to different users. In fact, comparing with Fig. 2 comparing the independent stream scheduler even outperforms the average throughput obtained using spatial multiplexing with optimal receivers. From the perspective of maximizing the system throughput using the scheduling disciplines considered herein, antennas are part of the shared spatial resource and should be divided appropriately between users on the downlink. Thus while spatial multiplexing offers high link capacity, in the multiuser environment there may be substantial degradation in scheduling packets to a single user on all transmit antennas.

7. CONCLUSION

In this paper we examined the multiuser diversity effect in MIMO communication systems. We assumed the use of spatial multiplexing and studied potential improvement in average throughput under various scheduling disciplines. Based on practical concerns we focused on spatial multiplexing systems that employ the zero-forcing linear receiver. We showed that even with these suboptimal receivers there can be substantial performance improvement due to multiuser diversity gain. We also showed that further improvements may be obtained by assigning antennas independently to different users and not all to a single user. Optimal scheduling algorithms for both partial and full channel knowledge at the transmitter for MIMO systems is a topic for future investigation.

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