Comparing the Performance of Coded Multiuser OFDM and Coded MC-CDMA over Fading Channels

Christian Ibars and Yeheskel Bar-Ness
Center for Communications and Signal Processing Research
New Jersey Institute of Technology
University Heights
Newark, NJ 07102 USA

Abstract — **The bit error rate of a coded Multiuser OFDM (M-OFDM) system and coded Multicarrier CDMA (MC-CDMA) system with multiuser detection is analyzed. The paper focuses on downlink wireless applications, where the channel suffers from frequency-selective fading. We investigate both systems when long time interleaving is possible, though introducing large delays, and when time interleaving is limited. Bit error rate lower bounds and simulation results show that, when the system load is high, M-OFDM performs similar to MC-CDMA while its implementation is less complex as it does not require multiuser detection.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a multicarrier modulation widely used in both wireless and wired applications. In the wireless arena, OFDM has been standardized for Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB) and Wireless Local Area Networks (WLAN). OFDM can effectively handle frequency-selective fading without complex equalization structures and the enhancement of noise at the receiver. The demodulation and modulation processes have very low complexity when the Fast Fourier Transform (FFT) and its Inverse (IFFT) are used.

Wireless communications share the transmission media, namely the wireless channel, and generally involve multiple users. Hence the need for a multiple access technique. There are different multiple access schemes based on OFDM modulation. We focus on two: Multiuser OFDM (M-OFDM) [1], also referred to as Orthogonal Frequency Division Multiple Access (OFDMA), and Multicarrier CDMA (MC-CDMA), as described in [2] and [3].

The performance of M-OFDM and MC-CDMA without channel encoding is well known (see [4] for the latter). On a frequency-selective channel MC-CDMA achieves frequency diversity by transmitting each user's symbols over all subcarriers. M-OFDM cannot exploit channel diversity because each symbol is transmitted over a single subcarrier. In this situation MC-CDMA outperforms M-OFDM.

The performance of coded MC-CDMA and coded OFDM is explored in [5], where an OFDM system with time division multiple access is considered. In this paper we consider M-OFDM and MC-CDMA for different system loads. By using channel encoding we can achieve frequency diversity with M-OFDM by means of frequency interleaving.

Doing so, different symbols of a codeword are transmitted over different subcarriers where fading is weakly correlated and large coding and diversity gain is achieved. A periodic interleaver has been suggested in [6] to maximize the code diversity. The coding gain in coded MC-CDMA is smaller since the diversity gain is already achieved by spreading. In this paper we present a performance analysis that determines which scheme offers lower bit error rate (BER) after channel coding.

In the next section, we describe the system models used in the comparison. In sections III and IV we give expressions for a lower bound on the BER and approximate the BER of M-OFDM and MC-CDMA. Simulation results and conclusions are presented in sections V and VI.

II. System model

The use of OFDM is the common part of the systems under study. We make use of a multicarrier channel model to represent the concatenation of the Inverse Fourier Transform, the frequency-selective fading channel and the Fourier Transform. The model is equivalent to having an independent subchannel for each subcarrier f_i provided that transmitter and receiver are perfectly synchronized. We assume that the subcarriers are narrowband and model each subchannel as a frequency-flat fading channel. Residual ISI resulting from this approximation can be eliminated using a guard interval to transmit a symbol prefix. Other assumptions that are made throughout the paper are perfect channel state information (CSI) at the receiver, no CSI at the transmitter, perfect synchronization, which eliminates inter-carrier interference, and absence of other cell interference (or alternatively it is modeled as Gaussian noise). We represent the L subcarrier fading channel as a diagonal matrix $\mathbf{A}$ with L correlated complex Gaussian-distributed coefficients $\{\alpha_1(n) \cdots \alpha_L(n)\}$ on the diagonal. We use the

0-7803-7206-9/01/$17.00 © 2001 IEEE

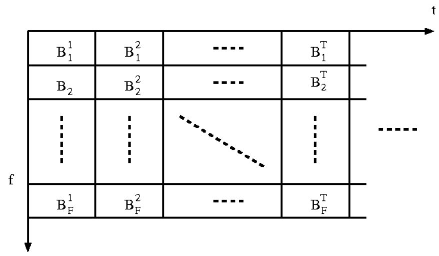

Figure 1: Multicarrier block fading channel

multicarrier block fading channel [6] to model the channel response. In this model, the transmitted sequence is divided in blocks B_j^i of time length τ symbols and frequency width f subcarriers, as in figure 1. The block size τ and f depend on the channel coherence time and coherence bandwidth respectively. The fading coefficients satisfy:

- $\alpha_i(n_1) = \alpha_j(n_2)$ if they are in the same block.
- $\alpha_i(n_1)$ and $\alpha_j(n_2)$ are independent otherwise.

The number of blocks along the frequency axis, F, is limited by the coherence bandwidth B_c and the signal bandwidth B_x to $F = \lceil \frac{B_x}{B_c} \rceil$, where $\lceil x \rceil$ denotes the smallest integer larger than x. The number of channel blocks available for interleaving is given by the product FT, T being the number of blocks along the time axis. T may be limited by delay constraints or transmission packet length. We look into two typical situations: first, we take T infinitely large. This corresponds to a transmission with no delay constraints (i.e. data transmission). With periodic interleaving, consecutive symbols are transmitted through independent channel states. The resulting channel is known as *independent fading channel.* Next we consider a situation where T is limited to a few blocks, corresponding to delay-sensitive applications or short data packets, where only FT channel states are available. The availability of more channel blocks allows the code to achieve higher diversity. The M-OFDM and MC-CDMA systems can be described with block diagrams as shown in figures 2 and 3 respectively. BPSK modulation is assumed in both cases.

In a M-OFDM system, different users $1 \cdots K$ are allocated subcarriers $f_1 \cdots f_K$. More generally, each user may be allocated a group of subcarriers.

We assume one subcarrier per user for simplicity. Before channel encoding, users' symbols are interleaved in time and frequency over all subcarriers $(f_1 \cdots f_L)$ to exploit the channel diversity. A periodic interleaver in time and frequency is used. Up to FT of each user's consecutive symbols are sent over a different channel block, and the sequence of blocks repeats periodically.

In a Multicarrier CDMA system, different users are assigned different spreading sequences $\mathbf{s}_k$. Spreading is performed only in the frequency domain, and each user symbol is sent over all carriers. We denote the spreading operation as

$$\mathbf{x}_k(n) = \mathbf{s}_k c_k(n), \tag{1}$$

where $\mathbf{x}_k(n) = [x_k^1(n) \cdots x_k^L(n)]^T$ is the signal sent through the channel at time n ($x_k^i(n)$ is sent over the i^{th} subcarrier) and $c_k(n)$ is k^{th} user's convolutionally encoded and BPSK modulated signal. The notation can be generalized to K users as

$$\mathbf{x}(n) = \mathbf{S}\mathbf{c}(n), \tag{2}$$

where each column of matrix $\mathbf{S}$, of size $L \times K$, and each element of $\mathbf{c}$ corresponds to one user. The processing gain is determined by L, the number of carriers used in the operation. The use of orthogonal spreading sequences does not eliminate multiuser interference when the channel is frequency-selective. We denote the correlation matrix of the channel-distorted sequences as

$$\mathbf{R_S(n)} = \mathbf{S}^*\mathbf{A}^*(n)\mathbf{A}(n)\mathbf{S}. \tag{3}$$

Here $*$ denotes a Hermitian transformation. To exploit time diversity, each user's signal is time-interleaved using a periodic interleaver over the time blocks.

III. BER LOWER BOUND ON M-OFDM

We denote the received signal for the n^{th} symbol of user k as

$$y_k(n) = \alpha_k(n)c_k(n) + n(n), \tag{4}$$

where $c_k(n)$ denotes the code symbol sent at time n, $\alpha_k(n)$ is the channel state for that symbol (to simplify notation, the subindex on the channel state $\alpha(n)$ refers to a subcarrier, a user or an enumerator of the different channel states, depending on the context), and $n(n)$ is white Gaussian noise. Denote two different codewords as $\mathbf{c}_i$ and $\mathbf{c}_j$ (at this point we do not need to differentiate between block and convolutional codes, but we assume that the code is linear).Using a maximum likelihood soft decoder, the pairwise error event probability (PEP) conditioned on the channel states is

$$P(\mathbf{c}_i \to \mathbf{c}_j|\alpha_i) = Q\left(\sqrt{\frac{d_E^2}{2N_0}}\right), \tag{5}$$

where $Q(x)$ denotes the complementary Gaussian cumulative distribution function, R_c the code rate, N_0 the noise spectral density and d_E^2 the squared Euclidean distance, conditioned on the channel state.

For infinitely large T the squared Euclidean distance is

$$d_E^2 = ||\mathbf{c}_i - \mathbf{c}_j||_2^2 = 4E_bR_c\sum_{i=1}^{d_H}|\alpha_i|^2, \tag{6}$$

where E_b denotes the bit energy, d_H Hamming distance, and α_i the channel coefficients multiplying symbols where the two codewords differ. α_i are independent complex Gaussian random variables with unit variance. We define

$$\gamma_O = \sum_{i=1}^{d_H} |\alpha_i|^2 \sim \mathcal{X}^2_{2d_H}, \quad (7)$$

where $\mathcal{X}^2_{2dH}$ denotes a chi-square distribution with $2d_H$ degrees of freedom. The average error event probability is given by

$$P(\mathbf{c}_i \to \mathbf{c}_j) = E_{\gamma_o} Q\left(\sqrt{\frac{2E_b}{N_0} R_c \gamma_o}\right), \quad (8)$$

where E_{γ_o} is the expectation operator. Equation (8) can be computed in closed form as ([7], p. 781)

$$P(\mathbf{c}_i \to \mathbf{c}_j) = \left(\frac{1-\mu}{2}\right)^{d_H} \sum_{k=0}^{d_H-1} \binom{d_H - 1 + k}{k} \left(\frac{1+\mu}{2}\right)^k, \quad (9)$$

with

$$\mu = \sqrt{\frac{E_b R_c / N_0}{1 + E_b R_c / N_0}}. \quad (10)$$

A lower bound on the bit error rate is given by the probability of the minimum distance error event, P_{dmin}, corresponding to the error event between two codewords separated by the minimum Euclidean distance, which for BPSKcorresponds to the minimum Hamming distance d_{Hmin}. This lower bound can be obtained from (9) by replacing d_H by d_{Hmin}. For high SNR, the BER can be approximated by

$$P_b \sim n P_{dmin}, \quad (11)$$

where n is the number of codewords at the minimum Hamming distance.

For limited time diversity (i.e. a small value for T) the squared Euclidean distance between $\mathbf{c}_i$ and $\mathbf{c}_j$ is

$$d_E^2 = ||\mathbf{c}_i - \mathbf{c}_j||_2^2 = 4E_b R_c \sum_{i=1}^{\delta} l_i |\alpha_i|^2, \quad (12)$$

where the summation is over the channel states multiplying symbols in which $\mathbf{c}_i$ and $\mathbf{c}_j$ differ and l_i is the number of symbol discrepancies on channel state α_i. Note that the sumation is generally not a chi-square random variable. The minimum number of channel states in which any two codewords differ, δ_{min}, is known as the diversity of the code. Achievable code diversities for a number of codes are found in [8] in the context of block fading channels and the results are applicable in our case as well, using time and frequency interleaving. δ_{min} is upper-bounded by the minimum Hamming distance of the code d_{Hmin} and the available diversity of the channel, FT, and is readily found with the code trellis and the interleaving pattern. It is shown in [8] that δ_{min} is a dominant factor in BER calculation at high SNR. A lower bound on the BER is given by the PEP between the codewords with minimum pairwise diversity, δ_{min},

$$P_b^{FT} \geq E_{\alpha_1^2 \cdots \alpha_{\delta_{min}}^2} Q\left(\sqrt{\frac{2E_b}{N_0} R_c \sum_{i=1}^{\delta min} l_i |\alpha_i|^2}\right). \quad (13)$$

Equation (13) can be evaluated using numerical integration. An approximation for the BER at SNR is

$$P_b^{FT} \sim n P_{dmin}^{FT}, \quad (14)$$

where P_{min}^{FT} is the PEP of the minimum distance error event, given parameters F and T.

IV. BER lower bound on MC-CDMA

As described in section II, MC-CDMA suffers from multiuser interference due to the loss of orthogonality between users' spreading sequences when they are transmitted over a frequency-selective channel. Two multiuser detectors are considered for the analysis of MC-CDMA: a decorrelating detector and a Minimum Mean Square Error (MMSE) detector [9]. We denote user k received signal, after the spreading sequence matched filters, by

$$\begin{aligned} y_k(n) &= (\mathbf{S}^*\mathbf{A}^*(n)\mathbf{A}(n)\mathbf{S}\mathbf{c}(n))_k + (\mathbf{S}^*\mathbf{A}^*(n)\mathbf{n}(n))_k \\ &= (\mathbf{R_S}(n)\mathbf{c}(n))_k + n'_k(n), \end{aligned} \quad (15)$$

where $\mathbf{n}(n)$ is a vector of white Gaussian noise with covariance matrix $R_n = \sigma_n^2 \mathbf{I}$.

A Decorrelating detector

The decorrelating detector eliminates all multiuser interference by multiplying the vector of matched filter outputs by the inverse of $\mathbf{R_S}(n)$. The detector output for user k is then

$$y_k(n) = c_k(n) + n''_k(n), \quad (16)$$

where $n''_k(n)$ is Gaussian noise with variance

$$\sigma^2_{n''} = \sigma_n^2 (\mathbf{R_S}^{-1})_{kk}(n). \quad (17)$$

$(\mathbf{R_S}^{-1})_{kk}$ is called *noise enhancement factor*. We drop the symbol index n to simplify notation. The noise enhancement factor depends on the spreading sequences and the channel response. We define the modified squared Euclidean distance between $\mathbf{c}_i$ and $\mathbf{c}_j$ as

$$d^2_{EmD} = 4E_b R_c \sum_{i=1}^{d_H} (\mathbf{R_S}^{-1})^{-1}_{kk}(i) = 4E_b R_c \gamma_D, \quad (18)$$

where we have incorporated the inverse of the noise enhancement factor as a multiplying factor and defined the parameter γ_D. The BER of MC-CDMA can be analyzed in a similar way to M-OFDM. A lower bound on the BER

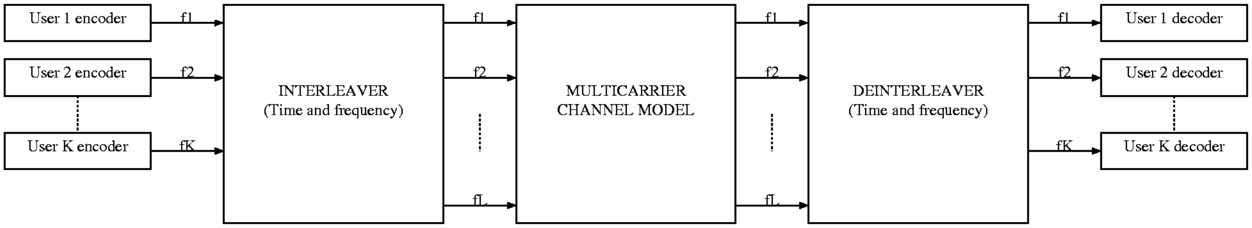

Figure 2: M-OFDM transmitter/receiver.

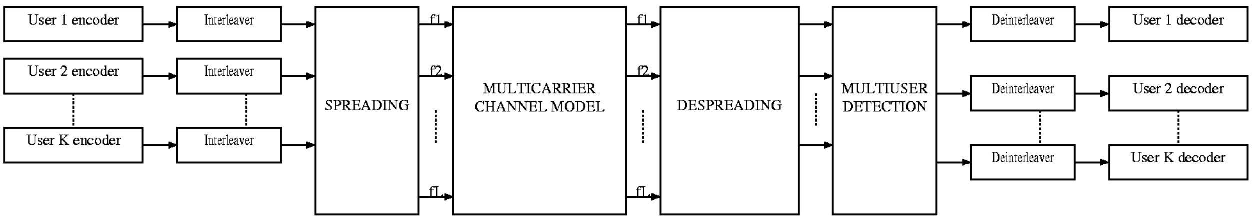

Figure 3: MC-CDMA transmitter/receiver.

is given by the pair of codewords having smallest pairwise diversity,

$$P_b^{FT} \geq E_{\alpha_1^2\cdots\alpha_{FT}^2} Q\left(\sqrt{\frac{2E_b}{N_0}R_c\sum_{i=1}^{d_{Hmin}}(\mathbf{R_S}^{-1})_{kk}^{-1}(i)}\right) \quad (19)$$

for infinitely large T and

$$P_b^{FT} \geq E_{\alpha_1^2\cdots\alpha_{FT}^2} Q\left(\sqrt{\frac{2E_b}{N_0}R_c\sum_{i=1}^{\delta_{min}}l_i(\mathbf{R_S}^{-1})_{kk}^{-1}(i)}\right) \quad (20)$$

for limited time diversity T. Here N_0 is the noise spectral density before the detector. In MC-CDMA, the code diversity δ_{min} is upper-bounded by d_{Hmin} and T rather than FT since we do not use frequency interleaving. The diversity benefit obtained from the code is thus smaller and so is the coding gain. We can approximate the BER at high SNR as

$$P_b^{FT} \sim nP_{dmin}^{FT}. \quad (21)$$

B MMSE detector

The MMSE detector multiplies the received signal vector by matrix

$$\mathbf{M}(n) = (\mathbf{R}_s(n) + \sigma^2\mathbf{I}_K)^{-1}. \quad (22)$$

The detector output for user k is

$$\begin{aligned} y_k &= (\mathbf{MR_s c})_k + n_k'' \\ &= (\mathbf{MR_s})_{kk}c_k + \sum_{i=1,i\neq k}^{K}(\mathbf{MR_s})_{ki}c_i + n_k'', \end{aligned} \quad (23)$$

where the first term corresponds to the desired user signal, the summation in the second term corresponds to multiuser interference, and the last term is Gaussian noise with covariance

$$\sigma_{n''}^2 = (\mathbf{MR_s M})\sigma^2, \quad (24)$$

where σ^2 is the noise power before the detector. We have omitted the symbol time index n to simplify notation. The multiuser interference term in (23) is well approximated by a Gaussian random variable I of variance

$$\sigma_I^2 = 4E_bR_c\sum_{i=1,i\neq k}^{K}(\mathbf{MR_S})_{ki}^2. \quad (25)$$

From (23) the squared Euclidean distance between two codewords separated by d_H symbols is given by

$$d_E^2 = 4E_bR_c\sum_{i=1}^{d_H}\left((\mathbf{MR_s})_{kk}(i)\right)^2, \quad (26)$$

where the index i is over the symbols where $\mathbf{c}_i$ and $\mathbf{c}_j$ are different. For infinitely large T, and incorporating the noise enhancement factor as we did in (18), we can define the modified squared Euclidean distance as

$$d_{EmM}^2 = 4E_bR_c\sum_{i=1}^{d_H}\frac{\left((\mathbf{MR_s})_{kk}(i)\right)^2}{\mathbf{MR_S M}(i) + \sigma_I^2(i)/N_0}, \quad (27)$$

where N_0 is the noise spectral density before the detector. The average PEP is given by

$$P(\mathbf{c}_i \rightarrow \mathbf{c}_j) = E_{\zeta_i}Q\left(\sqrt{\frac{2E_b}{N_0}R_c\sum_{i=1}^{d_H}\zeta_i}\right), \quad (28)$$

where

$$\zeta_i = \frac{(\mathbf{MR}_{kk}(i))^2}{\mathbf{MRM}_{kk}(i) + \sigma_I^2(i)/N_0} \tag{29}$$

and ζ_i are independent. For small T, (28) can be expressed as

$$P(\mathbf{c}_i \to \mathbf{c}_j) = E_{\zeta_i} Q\left(\sqrt{\frac{2E_b}{\sigma^2} R_c \sum_{i=1}^{\delta_d} l_i \zeta_i}\right) \tag{30}$$

An approximate lower bound on the BER is given from (28) and (30) by replacing d_H by d_{Hmin} and δ_d by δ_{dmin} respecively. Approximate perfomance at high SNR is obtained by

$$P_b \sim nP_{dmin}, \tag{31}$$

where P_{dmin} is the minimum distance PEP and n is the number of codewords at minimum distance.

V. Simulation results

BER simulations for a M-OFDM system and a MC-CDMA were carried out for a channel with diversity limited to $F = 4$ and $T = 2$. The results are shown in figure 4 for different SNR for a 64-state, rate 1/2 convolutional code with generators (133,171). No attempt was made to optimize the code for this particular diversity order FT. A fully loaded system with 32 different carriers, one carrier per user, was used. The lower bounds found in sections 3 and 4 are also shown in figure 4. We can see that M-OFDM and MC-CDMA/MMSE have approximately the same performance.

A partially loaded system has been considered with 32 carriers and a variable number of users for the 4-state code (7,5). Figure 5 shows the simulated BER and the BER lower bounds over the number of active users in the system. The performance of MC-CDMA is better with only a few users and deteriorates when the number of users increases. The performance of M-OFDM with a fixed coding scheme like the one used is insensitive to the number of users, but rate-adaptive coding schemes can be used to improve performance at low load.

VI. Conclusions

The performance of two alternatives for multiuser multicarrier communications has been studied for synchronous wireless systems. Multiuser OFDM exhibits a large coding gain and relies on the coding scheme to exploit the channel frequency diversity, whereas Multicarrier CDMA can achieve a certain diversity gain regardless of the channel code used and has a smaller coding gain. It is shown that performance depends on load, interleaving depth and diversity order, and that Multiuser OFDM matches the performance of MC-CDMA with a MMSE detector at high loads while outperforming the decorrelating detector.

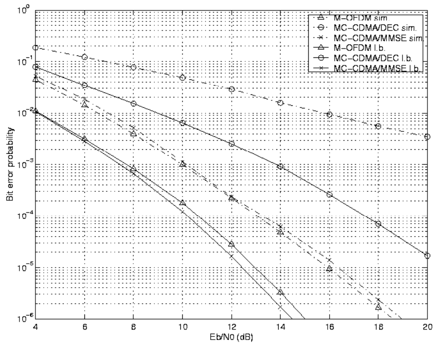

Figure 4: Bit Error Rate at full load. 32 users/32 carriers.

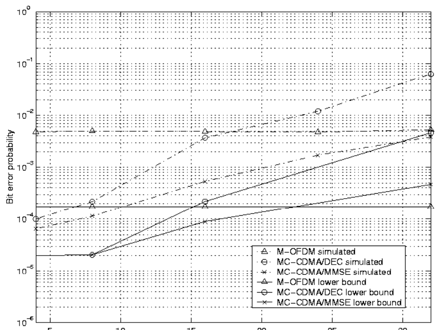

Figure 5: Bit Error Rate at partial load. $E_b/N_0 = 8dB$.

References

[1] E. Lawrey, "Multiuser OFDM," in *Proceedings ISSPA '99*, 1999, pp. 761–764.

[2] N. Yee, J.P. Linnartz, and G. Fettweis, "Multi-Carrier CDMA in indoor wireless radio networks," in *IEEE PIMRC International Conference*, 1993, pp. 109–113.

[3] M. A. Visser and Y. Bar-Ness, "Adaptive reduced complexity Multicarrier CDMA (MC-CDMA) structure for downlink PCS," *European Trans. on Telecommunications*, vol. 10, no. 4, pp. 1–10, August 1999.

[4] S. Hara and R. Prasad, "Design and performance of Multicarrier CDMA system in frequency-selective Rayleigh fading channels," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 5, pp. 1584–1594, September 1999.

[5] R. Le Gouable and M. Helard, "Performance of MC-CDMA systems in multipath indoor environments. Comparison with COFDM-TDMA system," in *3G Mobile Communication Technologies, First International Conference on (Conf. Publ. No. 471)*, 2000, pp. 81–85.

[6] R. Wesel and J. Cioffi, "Fundamentals of coding for broadcast OFDM," in *Proc. Asilomar Conf. '95*, 1995, pp. 2–6.

[7] J. G. Proakis, *Digital Communications*, McGraw-Hill, 1995.

[8] R. Knopp and P. Humblet, "On coding for block fading channels," *IEEE Trans. on Information Theory*, vol. 46, no. 1, pp. 189–205, January 2000.

[9] S. Verdu, *Multiuser Detection*, Cambridge University Press, 1998.