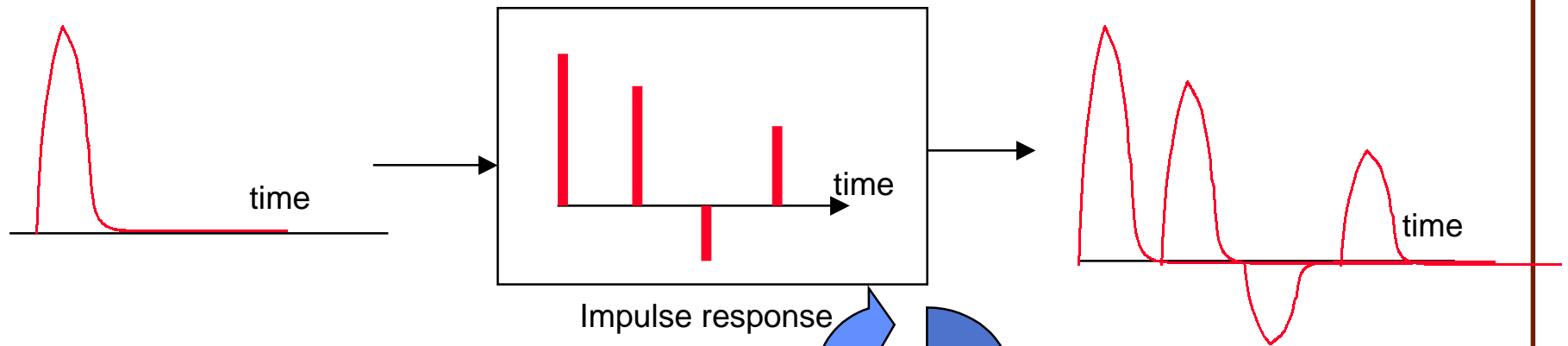
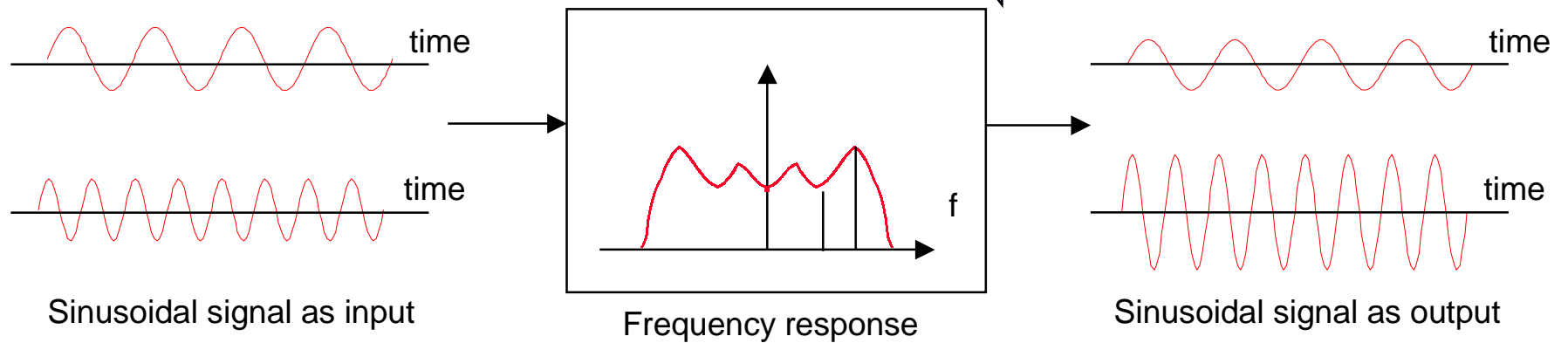


Multipath can be described in two domains: time and frequency

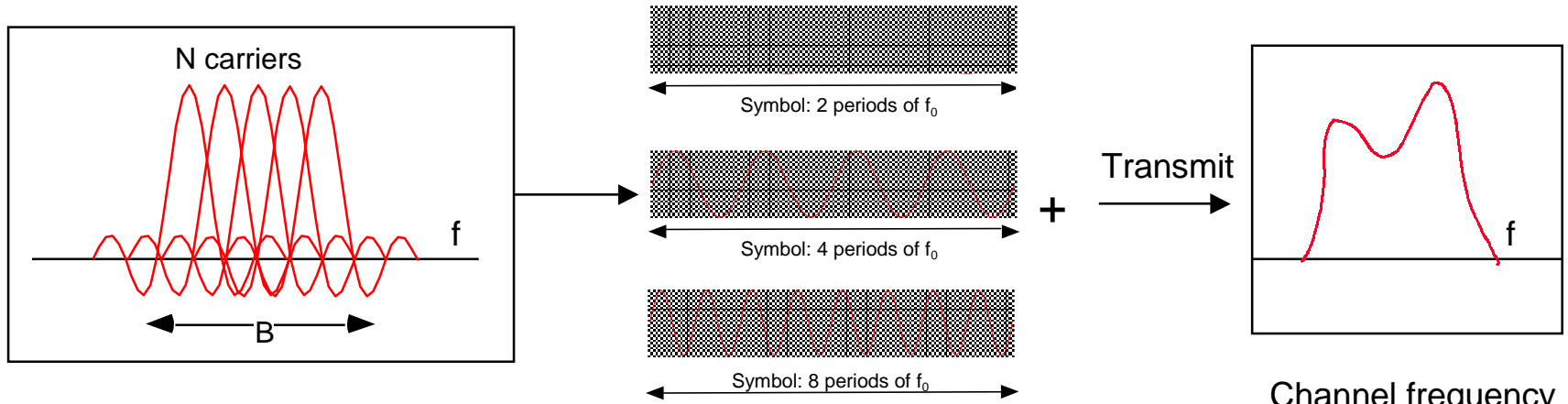
Time domain: Impulse response



Frequency domain: Frequency response



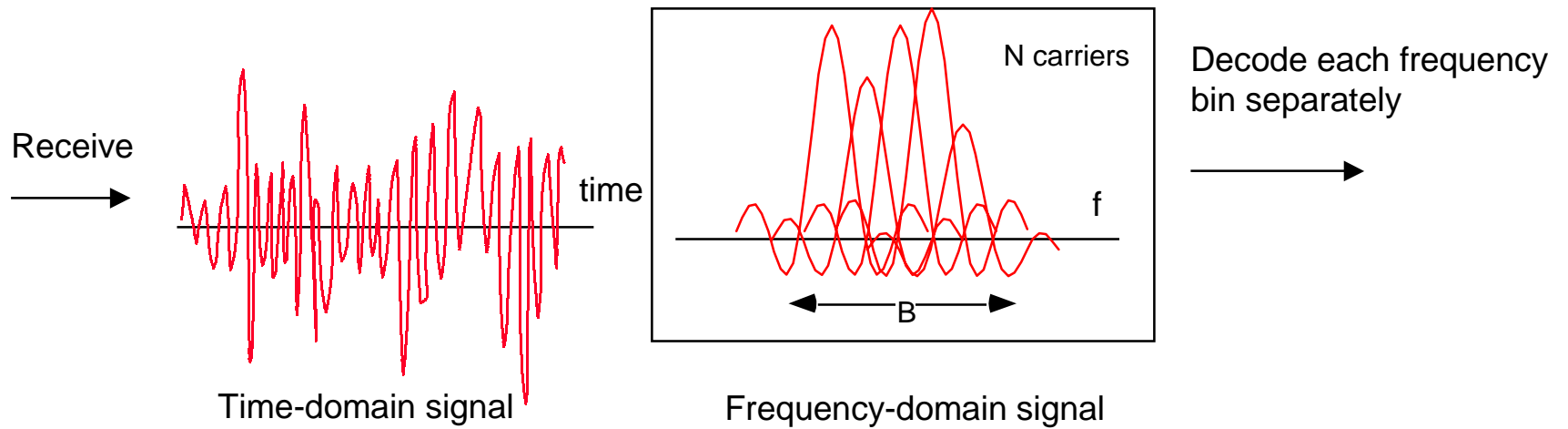
Introduction to OFDM modulation



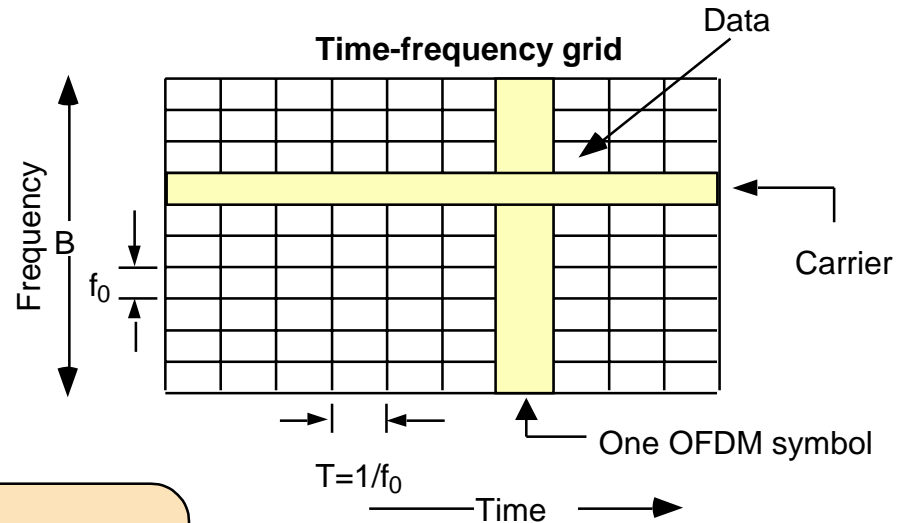
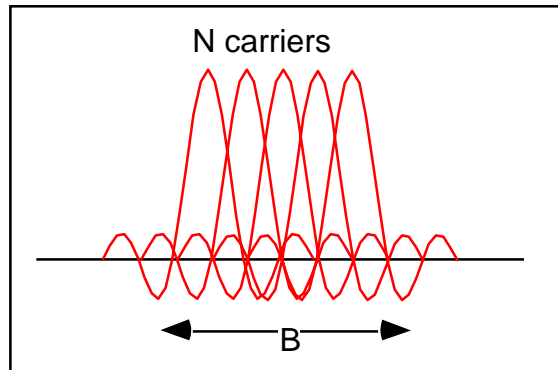
Data coded in frequency domain

Transformation to time domain:
each frequency is a sine wave
in time, all added up.

Channel frequency
response



Introduction to OFDM (Orthogonal frequency division multiplex)



Features

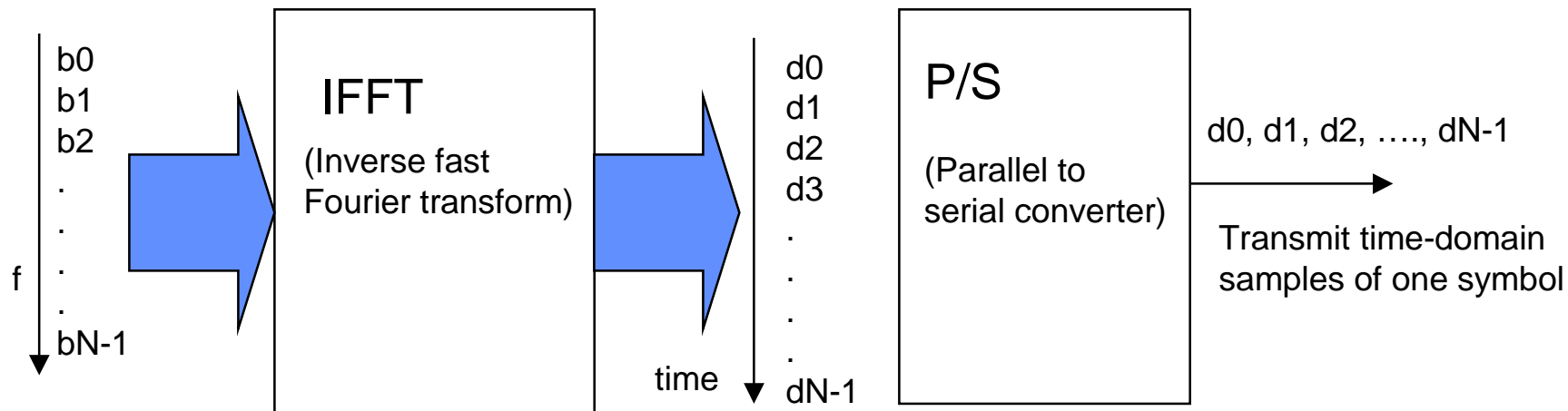
- No intercarrier guard bands
- Controlled overlapping of bands
- Maximum spectral efficiency (Nyquist rate)
- Easy implementation using IFFTs
- Very sensitive to time-freq. synchronization

**Intercarrier Separation =
Any integer Multiple of $1/(\text{symbol duration})$**

Modulation technique

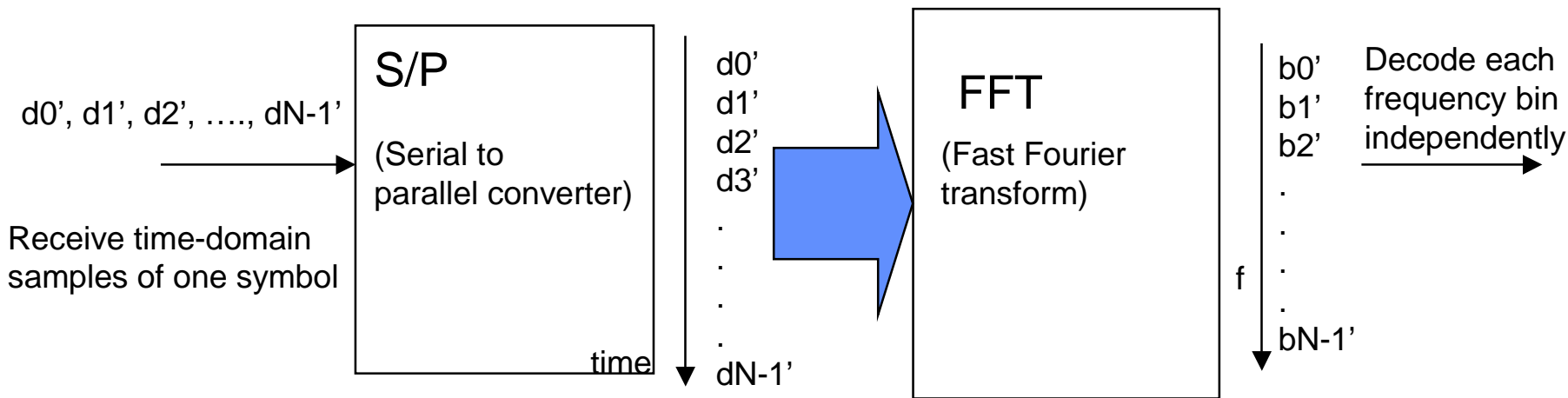
A user utilizes all carriers simultaneously to transmit its data as coded quantity at each frequency carrier, which can be quadrature-amplitude modulated (QAM).

OFDM Modulation and Demodulation using FFTs



Data coded in frequency domain: one symbol at a time

Data in time domain: one symbol at a time



Receive time-domain samples of one symbol

Decode each frequency bin independently

Loss of orthogonality (by frequency offset)

Transmission pulses

$$\psi_k(t) = \exp(jk2\pi t/T) \quad \text{y} \quad \psi_{k+m}(t) = \exp(j2\pi(k+m)t/T)$$

Reception pulse with offset δ

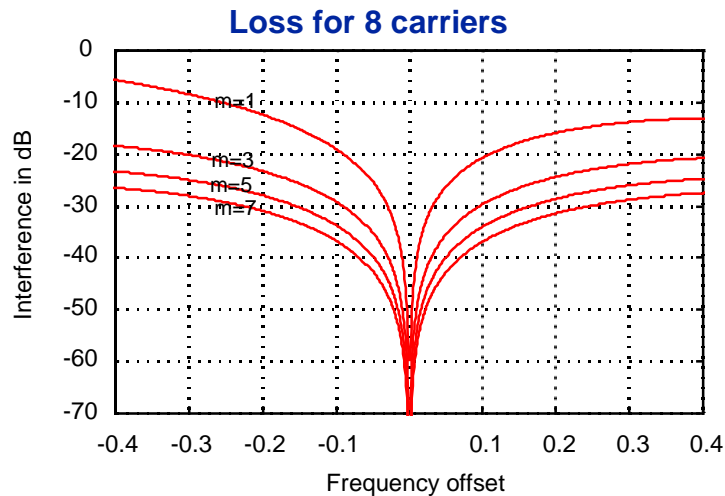
$$\psi_{k+m}^\delta(t) = \exp(j2\pi(k+m+\delta)t/T) \quad \text{con} \quad |\delta| \leq 1/2$$

Interference between channels k and k+m

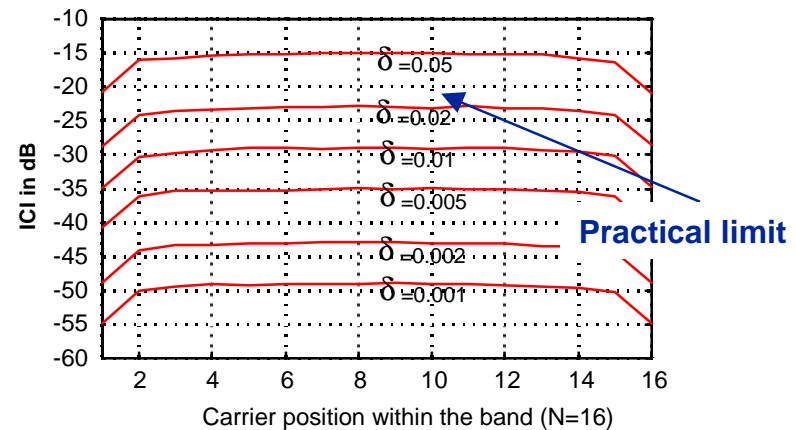
$$I_m(\delta) = \int_0^T \exp(jk2\pi t/T) \exp(-j(k+m+\delta)2\pi t/T) dt = \frac{T(1 - \exp(-j2\pi\delta))}{j2\pi(m+\delta)}$$

$$|I_m(\delta)| = \frac{T|\sin \pi\delta|}{\pi|m+\delta|}$$

Summing up $\forall m$ $\sum_m I_m^2(\delta) \approx (T\delta)^2 \sum_{m=1}^{N-1} \frac{1}{m^2} \approx (T\delta)^2 \frac{23}{14}$ for $N \gg 1$ ($N > 5$ is enough)



Total ICI due to loss of orthogonality



δ assumed r.v.
Gaussian $\sigma = \delta$

Loss of orthogonality (time)

Let us assume a misadjustment τ

$$X_i = c_0 \int_{-T/2}^{-T/2+\tau} \psi_k(t) \psi_l^*(t-\tau) dt + c_1 \int_{-T/2+\tau}^{T/2} \psi_k(t) \psi_l^*(t-\tau) dt$$

2 consecutive symbols

Then $|X_i| = \begin{cases} 2T \left| \frac{\text{sen } m\pi \frac{\tau}{T}}{m\pi} \right|, & c_0 \neq c_1 \\ 0, & c_0 = c_1 \end{cases}$
 if $m=k-l$



Or approximately, when $\tau \ll T$

$$\frac{|X_i|}{T} \approx \frac{2m\pi \frac{\tau}{T}}{m\pi} = 2 \frac{\tau}{T} \quad \text{independent on } m$$

In average, the interfering power in any carrier is

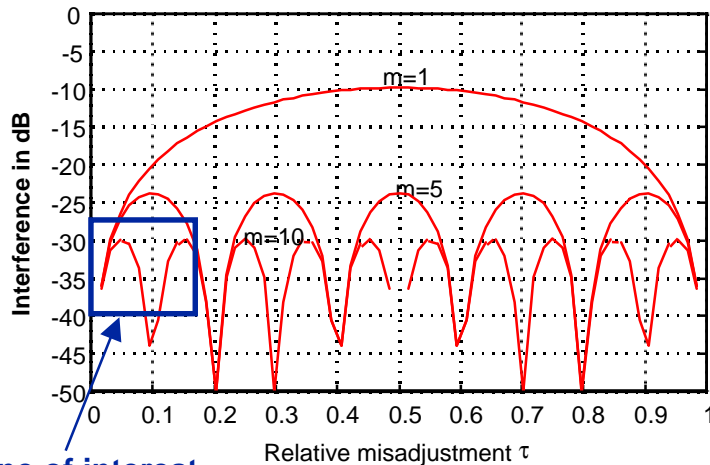
$$E\left[\frac{|X_i|^2}{T^2}\right] = 4\left(\frac{\tau}{T}\right)^2 \frac{1}{2} + 0 \frac{1}{2} = 2\left(\frac{\tau}{T}\right)^2$$



$$ICI \approx 20 \log\left(\sqrt{2} \frac{\tau}{T}\right), \quad \tau \ll T$$

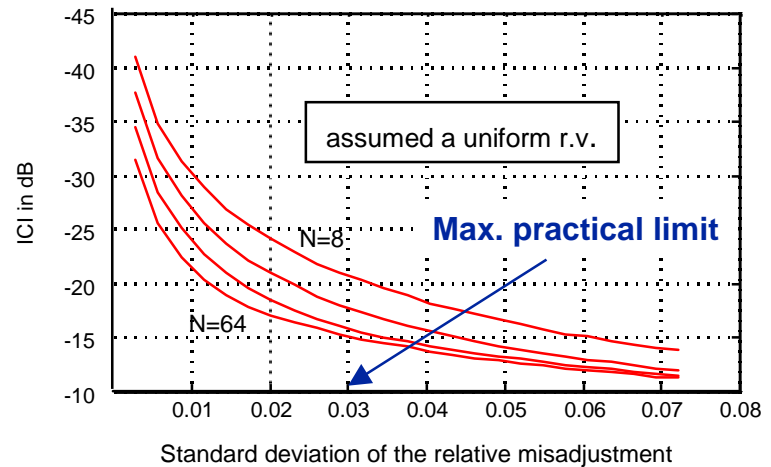
Per carrier

Loss for 16 carriers



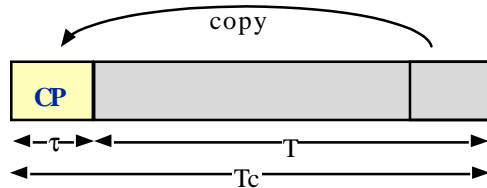
Zone of interest

ICI due to loss of orthogonality

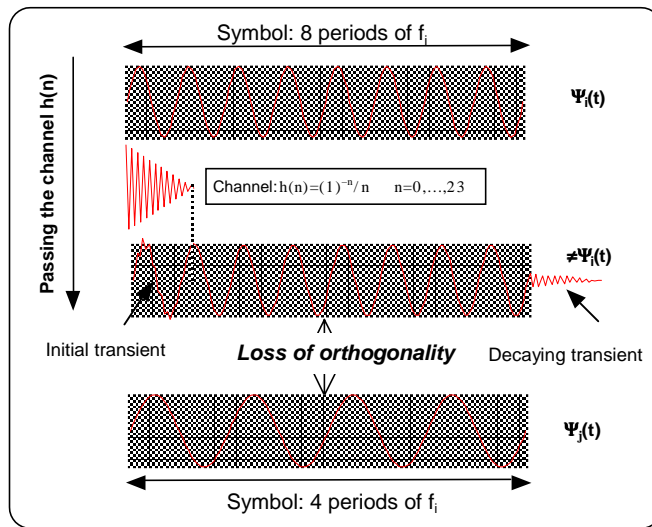


Including a cyclic prefix to each OFDM symbol

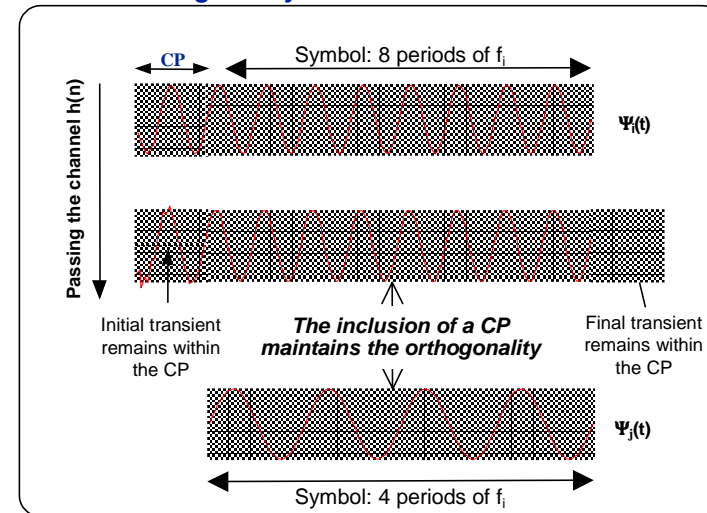
To combat the multipath: including time guards between the symbols



Without the Cyclic Prefix



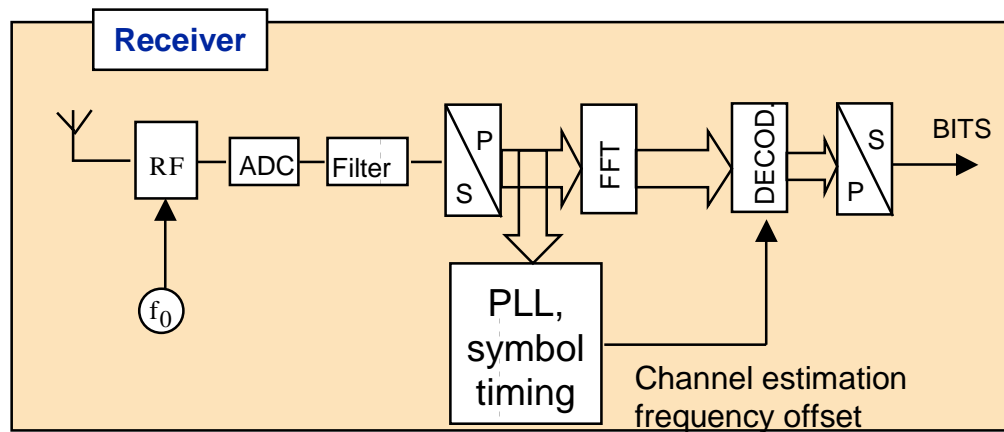
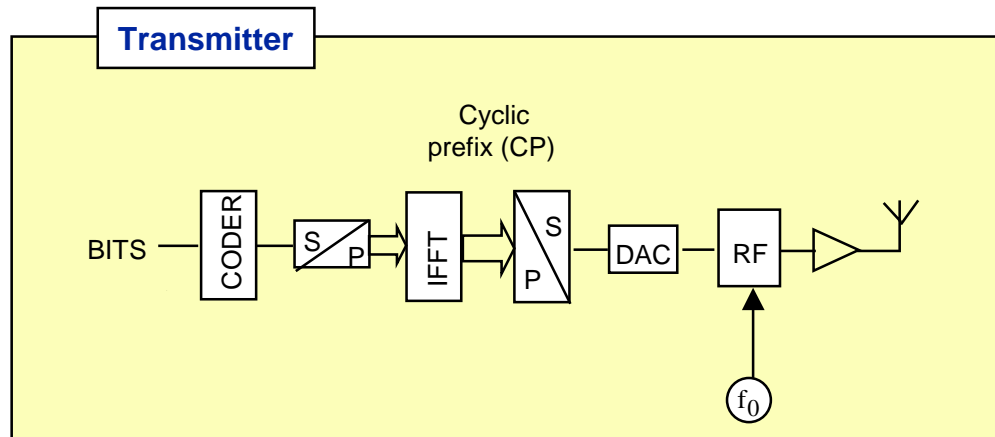
Including the Cyclic Prefix



CP functions:

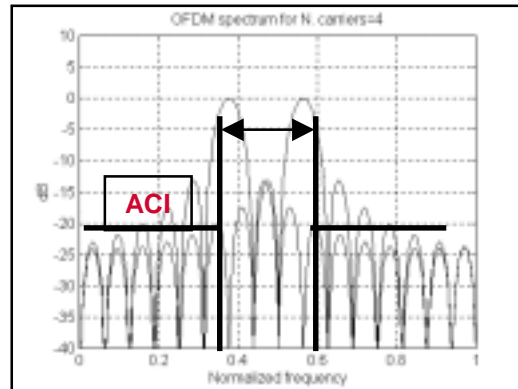
- It **acomodates** the decaying transient of the previous symbol
- It **avoids** the initial transient reaches the current symbol

Simplified scheme of an OFDM transceiver

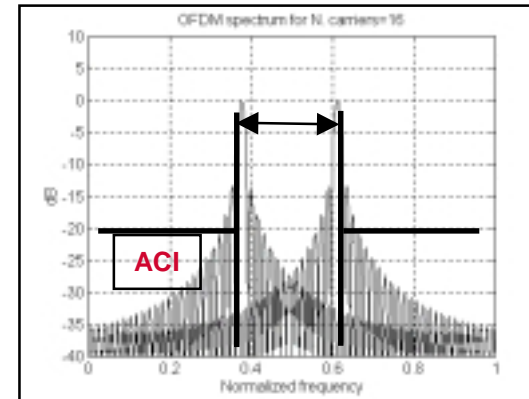


Windowing of the OFDM symbol

Total band used by OFDM: it depends on the number of carriers



Wide separation



Narrow separation

Maintaining a fix bandwidth, if **N increases** → **Adjacent channel interference decreases**

BUT

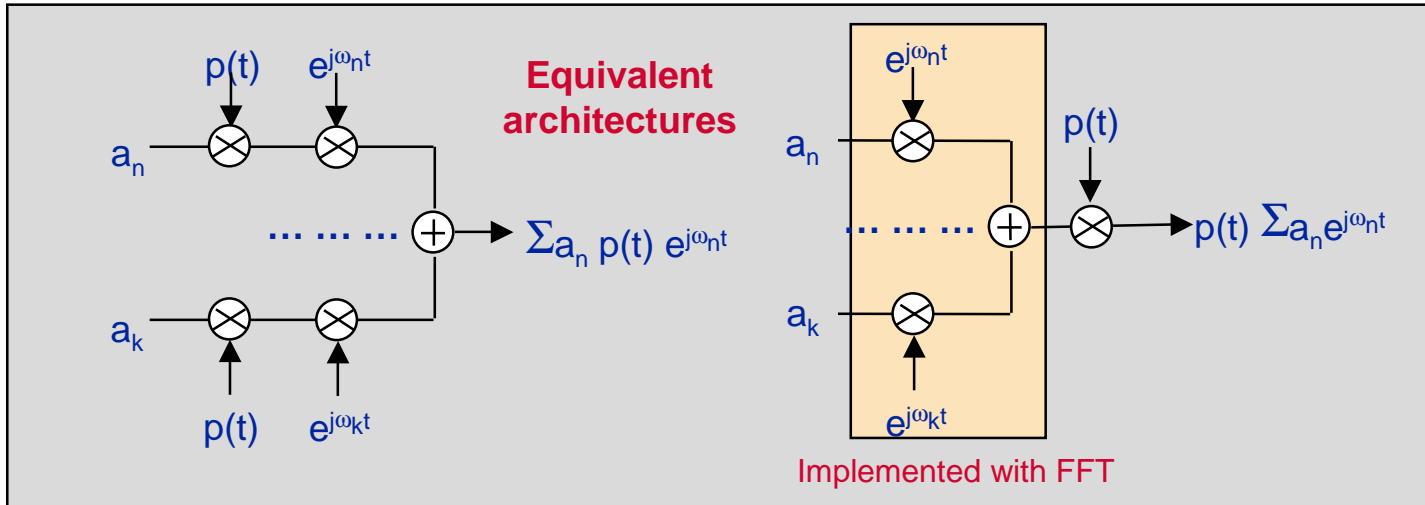
It is interesting to have **few** carriers as well:

- **To introduce short delay in data gathering and signal processing (FFTs)**
- **To have a bigger intercarrier separation --> It reduces the relative frequency offset**

Compromise

Need to shape the OFDM symbols

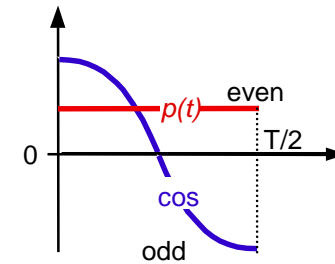
OFDM modulators with symbol shaping



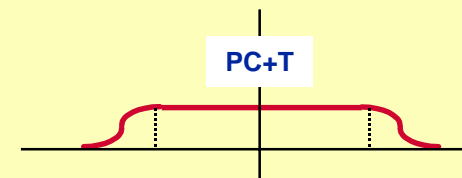
After the synchronous reception

$$I = \int_{-T/2}^{T/2} p(t) e^{j2\pi(k-n)t/T} dt \stackrel{p(t) \text{ even}}{\approx} 2 \int_0^{T/2} p(t) \cos[2\pi(n-k)t/T] dt = \begin{cases} 0, & k \neq n \\ 1, & k = n \end{cases}$$

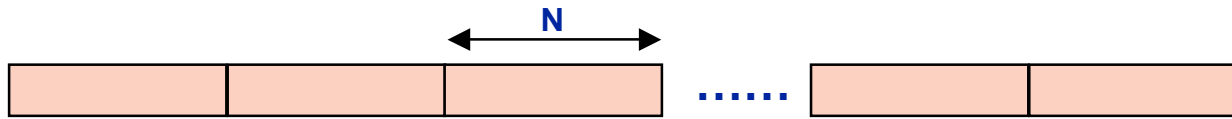
The simplest way to maintain symmetry within $-T/2 < t < T/2$ is $p(t) = k$



- ❑ Symbol shaping has to be carried out as part of the symbol duration + CP
- ❑ The total ACI can be considerably reduced

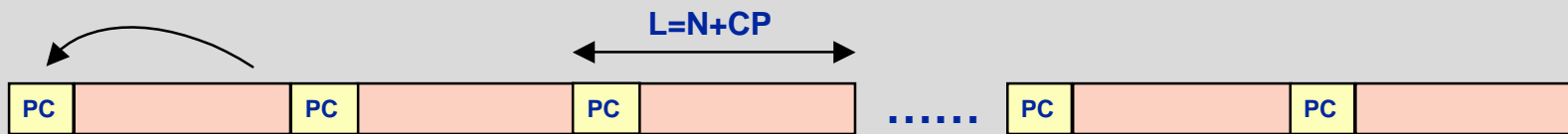


Robustness against the channel and ACI improvement



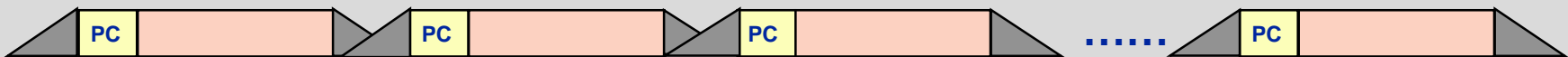
Virtual OFDM symbols within the slot

- With **guards** (Cyclic prefixes), the channel's **time dispersion is avoided**



OFDM symbols with time guards (CPs)

- With smooth transitions between symbols, the **adjacent channel interference** is minimized



OFDM symbols with time guards and symbol shaping

802.11a Physical Layer Data Symbol Format



Short training sequence: AGC and frequency offset
Long training sequence: Channel estimation

Training symbols: 4 us each

t: 0.8 us, 16 samples

GI2: 1.6 us, 32 samples

T: 3.2 us, 64 samples

Data Symbols: 4 us each

GI: 0.8 us, 16 samples

OFDM Symbol: 3.2 us, 64 samples

- * Only 52 of the 64 carriers are used.
- * 4 of the 52 carriers are used for pilot carriers (no data).

Data rate for each 20 Mhz channel:

20 Msamples per second.

250 Ksymbols per second.

48 data carriers per symbol.

1/2 or 3/4 convolutional code.

1 bit/carrier (BPSK) to 6 bits/carrier (64 QAM).

Overall:

Lowest: $48 * 1 * 1/2 * 250K = 6$ Mbps.

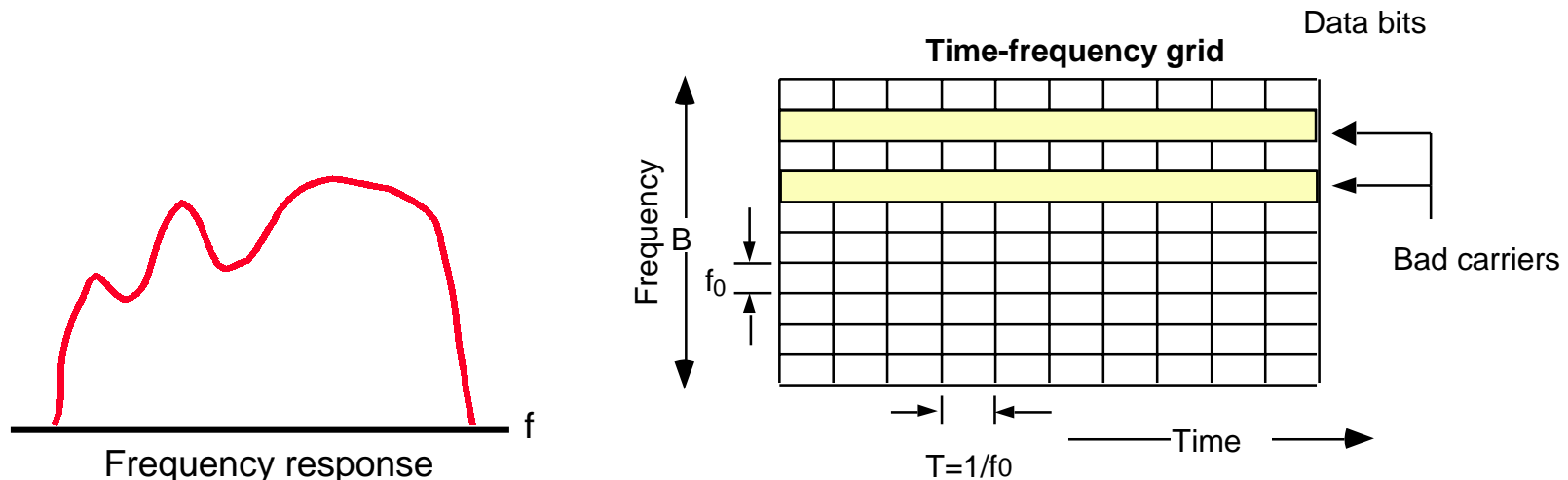
Highest: $48 * 6 * 3/4 * 250K = 54$ Mbps.

Turbo mode supports 108 Mbps using 40 Mhz channel.

Robustness against errors: random noise and channel-selected errors

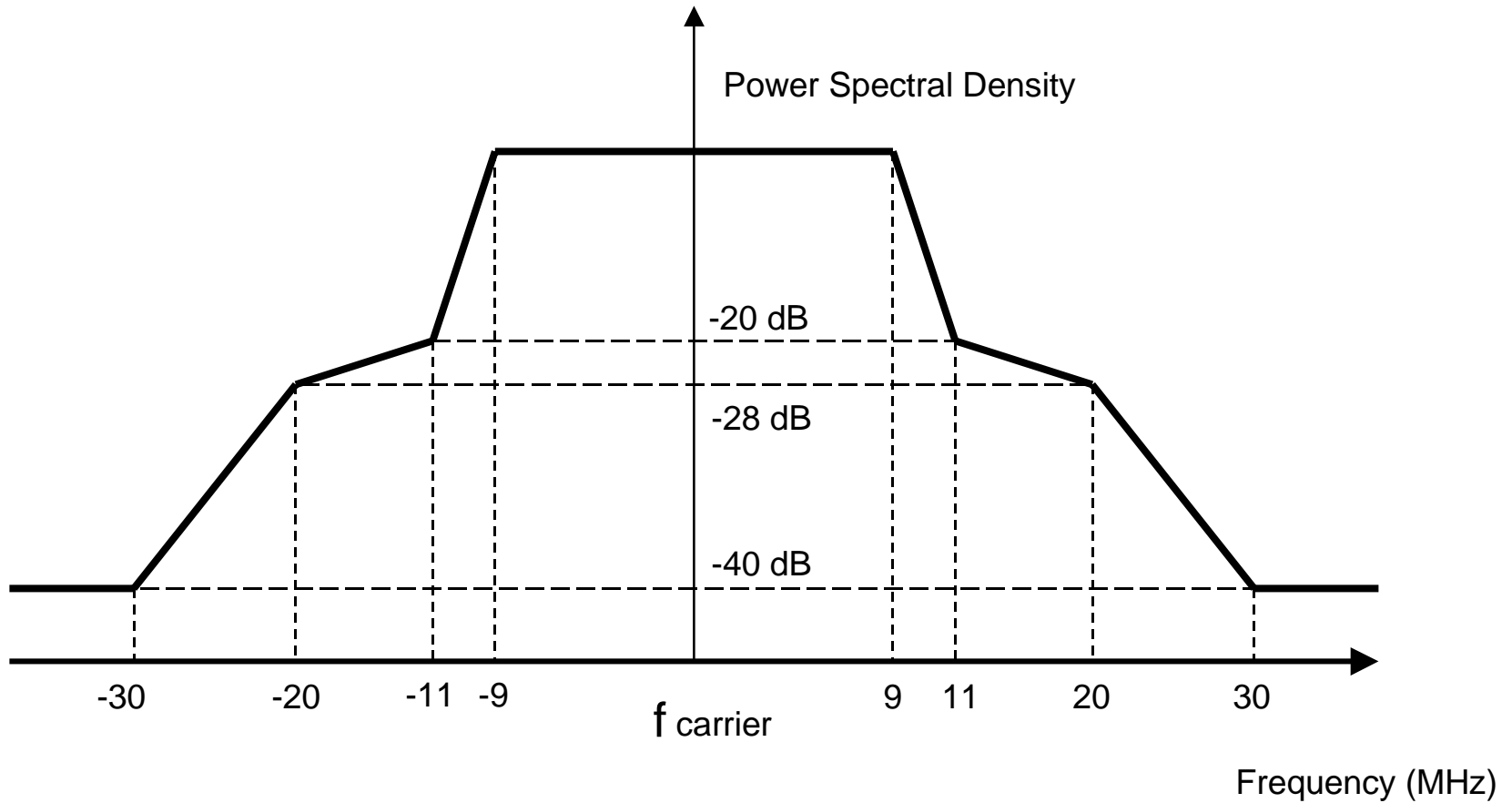
Random noise: primarily introduced by thermal and circuit noise.

Channel-selected errors: introduced by magnitude distortion in channel frequency response.



Errors are no longer random. Interleaving is often used to scramble the data bits so that standard error correcting codes can be applied.

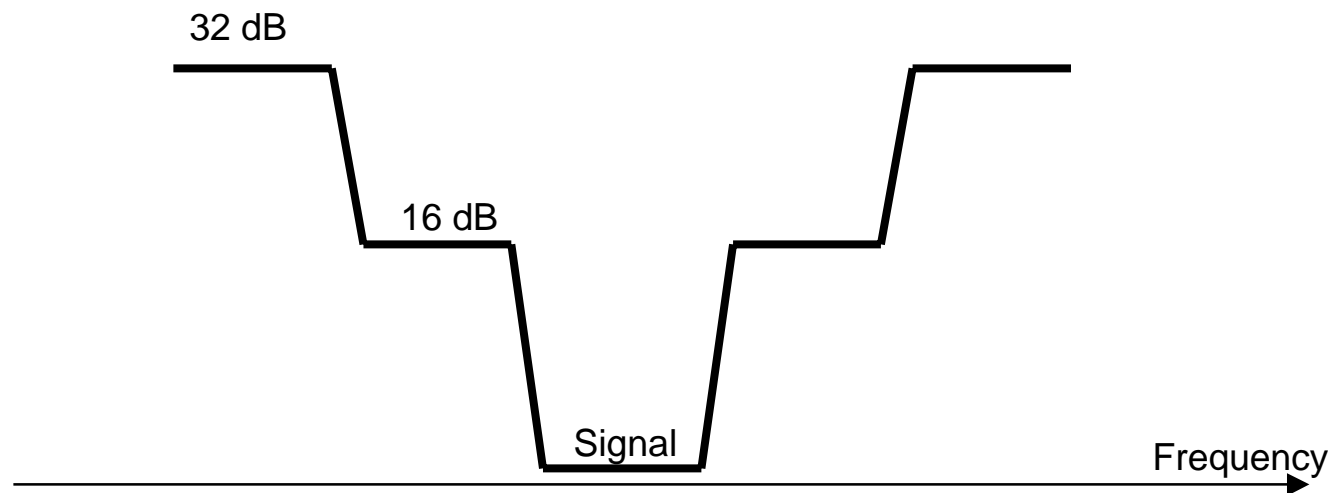
Spectrum Mask



- Requires extremely linear power amplifier design.

Adjacent Channel and Alternate Adjacent Channel Rejection

Data rate	Minimum Sensibility	Adjacent Channel Rejection	Alternate Channel rejection
6 M b p s	-82 d B m	16 d B	32 d B
12 M b p s	-79 d B m	13 d B	29 d B
24 M b p s	-74 d B m	8 d B	24 d B
36 M b p s	-70 d B m	4 d B	20 d B
54 M b p s	-65 d B m	0 d B	15 d B



- Requires joint design of the anti-aliasing filter and ADC.